

MOND WITH EINSTEIN'S COSMOLOGICAL TERM AS ALTERNATIVE TO DARK MATTER

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RESUMEN

Se postula un modelo cosmológico FRW con $\Lambda \equiv \Lambda(r)$, como alternativa a la materia oscura no bariónica. Este potencial se construye por reflexión especular del potencial de Yukawa: nulo dentro del sistema solar, poco atractivo en distancias interestelares, muy atractivo a rangos de distancia galácticos y repulsivo en escalas cósmicas. Este modelo es compatible con la densidad crítica observada, y con la teoría Milgrom.

ABSTRACT

It postulates a FRW cosmological model with $\Lambda \equiv \Lambda(r)$ as an alternative to the non-baryonic dark matter. This potential is build starting from a speculate reflection to Yukawa potential: zero in the inner solar system, slightly attractiveness in interstellar distances, very attractiveness in galactic distance ranges and repulsive to cosmic scales. This model is compatible with the density critical observed, and Milgrom theory

Key Words: cosmology — dark matter

1. INTRODUCTION

Assuming that the dynamics of the universe is prescribed only by the Newton gravity force we encounter serious difficulties in describing the Universe: it cannot explain the rotation curves of galaxies, the missing mass in rich clusters of galaxies and that the observed baryonic matter density is much lower than predicted by the FRW models with cosmological constant and zero curvature. The problem of missing mass appears to affect the dynamics at all length scales beyond the Solar System (Freese 2000). One solution has been to assume non-baryonic dark matter, however, its existence is only paradigmatic. Other alternatives are the modification of the Gravitation Universal Law to scales larger than the solar system, as MOND theories (Milgrom 2009). Although the formalism is lacking to connect these ideas with FRW models and the observables in Big Bang model. Remember that there is no experimental evidence to confirm the validity of Newtonian dynamics beyond the Solar System (Adelberger et al. 2003).

2. INVERSE YUKAWA FIELD (IYF)

We assume the existence of new fundamental interactions, whose origin is baryonic matter, similar to Newton gravity and which acts differently at different length scales, as did the approach of Yukawa for the strong interaction. This Yukawa type inverse

potential per unit mass, is built starting from a reflection to speculate of the Yukawa potential: null very near the solar system, slightly attractive at interstellar range distances, very attractive at distance ranges comparable to galaxies clusters and repulsive at cosmic scales:

$$U(r) \equiv U_0(M)(r - r_0)e^{-\alpha/r}, \quad (1)$$

where $U_0(M)$ is the magnitude that causes the field (in units of N/kg), α is an coupling constant of order of $2.5h^{-1}$ Mpc (the average value of an almost smooth transition distribution of galaxies to strong agglutination) and r_0 is of the order $50h^{-1}$ Mpc (the average distance between clusters of galaxies). As usual $H_0 = 100h$ km s⁻¹ Mpc is the Hubble constant. Figure 1 shows the variation of U/U_0 relative to the adimensional variable $x \equiv r/r_0$. We assume that any particle with nonzero rest mass is subject to the Newtonian gravitational force by the law of Universal Gravitation, and to an additional force that varies with distance, we call it the Inverse Yukawa field (IYF). Thus the gravitational force is bimodal (bigravity): it varies as the inverse square for $r \ll 1$ kpc, and it behaves in a very different manner when the comoving distance is of the order of kiloparsecs or larger.

We can see that in distance scales of the order of the Solar System this contribution is null; it is mildly attractive for distances of the order of kiloparsecs, strongly attractive at megaparsec distances, and repulsive at cosmological scales. Thus IYF, namely the bimodal complement large-scale Newtonian gravity is:

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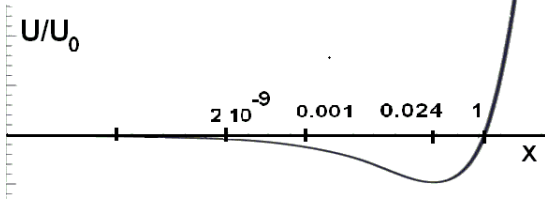


Fig. 1. Inverse Yukawa Potential in adimensional comoving scale $x = (r/50h^{-1} \text{ Mpc})$.

$$F_{YI}(r) \equiv -\frac{U_0(M)}{r^2} e^{-\alpha/r} (r^2 + \alpha(r - r_0)). \quad (2)$$

Also in the weak field approximation ($x \ll 1$) the IYF per unit mass is given by:

$$F_{YI}(r \ll r_0) \approx -\frac{U_0(M)\alpha r_0}{r^2}. \quad (3)$$

But if $x \rightarrow 0$, IYF is null, in accordance to Eotvos-type experiments. Is easy to see that for r the order of a kiloparsec, we recover the MOND-Milgrom assumptions (Milgrom 2009):

$$F_{YI}(r \ll r_0) \approx \frac{U_0(M)r_0}{2r + \alpha} \approx \left(\frac{U_0(M)r_0}{2}\right) r^{-1}. \quad (4)$$

Remember that the usual Newtonian gravitation acts in addition to this force. Notice that the maximum value of the force occurs at $r \approx 1.2h^{-1} \text{ Mpc}$, a typical Abell radius.

3. COSMOLOGICAL CONSEQUENCES

Let us now consider a usual homogeneous and isotropic FRW metric an usual energy-momentum tensor for a perfect fluid together with $\Lambda \equiv \Lambda(r)$, as dynamic variable proportional to IYF, thus:

$$\Lambda(r) = \Lambda_0(x - 1)e^{-\alpha_0/x}, \quad (5)$$

where Λ_0 is a coupling constant, and $\alpha_0 = 1/20$ is a dimensionless constant or $\alpha_0 = \alpha/r_0$. As before $x \equiv r/r_0$. Then $\Lambda_0 \approx 39H_0^2/c^2$ or $\Lambda_0 \approx 0.45h^2 10^{-50} \text{ m}^{-2}$. But now the definition of the critical density change is: $\rho_c \simeq 9.1 \frac{3H_0^2}{8\pi G} \approx 2.53 10^{12} h^2 M_{\text{sun}}/\text{Mpc}^3$.

When the critical density increases, because the critical mass has been underestimated in the usual definition, the IYF, must join the mass equivalent to the energy of the field, has to be taken into account. Also the central density in the core of clusters of galaxies is $3 \times 10^{15} M_{\text{sun}} \text{ Mpc}^{-3}$. For cosmological distance ranges, at scales larger than $50h^{-1} \text{ Mpc}$, the Λ variation is asymptotic (see Figure 1 for $x \gg 1$) and using equation (5) then

$$\Omega_\Lambda \approx \Lambda_0 \left(\frac{c^2}{3H_0^2}\right) \alpha_0 (x - 1). \quad (6)$$

Also, if we define: $\Omega_{YIF} \equiv -\frac{c^2 \Lambda(r=r_c)}{3H_0^2}$, then the Friedmann equation is (Falcon 2010):

$$\frac{kc^2}{R^2(t)} = H_0^2 [\Omega_m (1 + \Omega_{YIF}) + \Omega_\Lambda - 1], \quad (7)$$

where we used the standard notation (see Peacock 1999, for details) for the dimensionless parameters of density, cosmological “constant” and deceleration. Replacing equation (6) into equation (7) with $x \approx 2$, like comoving distance as $100h^{-1} \text{ Mpc}$, (cosmological distance range) then $\omega_\lambda \approx 0.65$ is very close to the usual value: 0.7.

4. CONCLUSION

The important result is that $k = 0$ and $\Omega_{YIF} \neq 0$ do not require the nonbaryonic dark matter assumption, i.e., using equation (7) and $\Omega_\lambda \approx 0.7$ we obtain $\Omega_m \approx \Omega_b = 0.03$ as the typical value for a flat universe but without nonbaryonic dark matter. For the early stages of the Universe, we find the usual relationship between $R(t)$ and the state variables ρ and P , furthermore do not affect the decoupling time, neither the predictions of the Cosmic Microwave Background, nor the primordial nucleosynthesis. Clearly, the incompatibility between the flatness of the Universe and $\Omega \ll 1$ is removed if brigravity is assumed, maybe like the IYF proposed here, as alternative to non-baryonic dark matter, also it is concomitant with FRW cosmology. Ishak et al. (2010) have shown that Λ , is a second order factor in the angle of deflection from gravitational lenses. It is clear that IYF also leads to a similar prediction. Also the IYF can fully comply the Mach Principle, through the incorporation of the dynamic cosmological term $\Lambda(r)$, which also implies that the masses of the nuclei of galaxies (Black Holes) have been overestimated, since the scalar field contributes in addition to the gravitational potential. At large distances from the sources, the reduction in the Newtonian field with the inverse of the square would be offset by an interaction that is growing at much greater distances. These long-range interaction also could be caused by baryonic mass and therefore could be derived with usual physics.

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